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Sensitivity of Multi Carrier 2 Dimensional Spreading Systems to Carrier Phase Noise

Youssef Nasser^(*), Mathieu des Noes^(*), Laurent Ros^(**) and Geneviève Jourdain^(**)

^(*) youssef.nasser@cea.fr / mathieu.desnoes@cea.fr CEA-LETI, 38054 Grenoble cedex 09 France

^(**) laurent.ros@lis.inpg.fr / genevieve.jourdain@lis.inpg.fr LIS, INPG, 46 Av. Felix Viallet, 38031 Grenoble France

Abstract—Phase noise is a topic of theoretical and practical interest in electronic circuits. Although progress has been made in the characterization of its description, there is still considerable gaps in its effect especially on the multi carrier spreading systems. In this paper, we investigate the impact of a local oscillator phase noise on the multi carrier 2 dimensional spreading systems known as OFDM-CDMA. The contribution of this paper is twofold. First, we use some properties of random matrix and free probability theory to give a simplified expression of the Signal to Interference and Noise Ratio SINR obtained after equalization and despreading. The latter is independent of the actual value of the spreading codes of different users and depends principally on the complex amplitudes of the estimated channel coefficients. Second, we use this expression to derive new weighting functions which are very interesting for the RF engineers when they design the frequency synthesizer. Simulation results are provided to discuss and validate our model.

Keywords: Multi carrier spreading systems, Large system analysis, Phase Noise, SINR.

I. INTRODUCTION

Recently, Orthogonal Frequency and Code Division Multiplexing (OFCDM) access technology has been investigated for the next generation of mobile communication systems [1][2]. It is a combination of Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA). To achieve high spectrum efficiency, these systems will implement a large number of sub-carriers and, as a consequence, will be highly sensitive to phase noise [3]. The phase noise creates two effects: the Common Phase Error CPE and the Inter Carrier Interference ICI. CPE alters equivalently the different carriers by causing sub carrier phase rotation while ICI introduces interferences to any sub carrier from all other sub carriers. In the literature, the phase noise effect has been presented in some papers for OFDM [3][4][5] and MC-CDMA systems [6]. Nevertheless, the analytical results of [6] have assumed independent identically distributed iid spreading codes i.e. the orthogonality between codes is not accounted for. Moreover, their results are only suitable for Additive White Gaussian Noise AWGN.

This paper presents the degradation introduced by a carrier phase noise on the performance of a downlink 2 dimensional spreading OFDM-CDMA system [1][2]. It extends to a 2 Dimensional OFDM-CDMA context the works of [7] which was done for a MC-CDMA system. It is also based on our previous works done for different

synchronization errors in OFDM-CDMA systems [8][9][10]. Using some properties of random matrix theory and free probability theory, a new analytical expression of the SINR has been developed. This SINR formula is independent of the actual values of the spreading codes while taking into account their orthogonality. It depends only on the complex amplitudes of the channel coefficients, the average transmitted power of the interfering users and the affected power of the desired user. It has been validated thanks to link level simulation based on accurate phase noise models [11]. Moreover, the SINR expression allows us to derive SINR degradations in different scenarios and to derive new weighting functions, such as proposed by Stott [4] for OFDM system and [7] for MC-CDMA systems. These functions will give a better understanding of the phase noise effect on the system performance.

This article is organized as follows. Section 2 describes the system model for OFDM-CDMA system including transmitter, channel model with carrier phase noise and receiver. Section 3 describes the phase noise model. Section 4 gives an asymptotical expression of the estimated SINR. In section 5, we derive new weighting functions for OFDM-CDMA systems while section 6 presents system performance. Eventually, conclusions are drawn in section 7.

II. SYSTEM MODEL

In this section, the generalized framework describing an OFDM-CDMA system is presented. Figure 1 shows a 2 dimensional (2D) spreading OFDM-CDMA transmitter/receiver chain for a downlink communication with N_u users [1]. For each user, each symbol is first spread by a Walsh-Hadamard WH sequence of N_c chips, and scrambled by a portion of N_c chips of the cell specific long pseudo random sequence. This scrambling code is used to minimize the multi-cell interference. The resulting chips are then allocated on the time/frequency grid as shown on Figure 2. Assuming an IFFT of N sub carriers, each user transmits $S=N/N_F$ data symbols $a_m[s]$ of the m^{th} user on the i^{th} OFDM-CDMA block composed of N_T OFDM symbols. N_F and N_T are respectively the frequency and time domain spreading factors. The system is thus a general case of the MC-CDMA [13] and MC-DS-CDMA [14] systems. The scheme is identical for other users using one specific WH sequence by user, with $N_u \leq N_c$. For simplicity and without loss of generality, we will drop the index i in the sequel.

Eventually, the signal at the IFFT input is:

$$b_q[sN_F + n] = \sum_{m=0}^{N_u-1} \sqrt{P_m} a_m[s] C_{m,s}[nN_T + q] \quad (1)$$

$$q = 0, \dots, N_T - 1; s = 0, \dots, S - 1, n = 0, \dots, N_F - 1$$

s is the index referring to the sub band used for the transmission of the symbol $a_m[s]$ of the m^{th} user. P_m is its transmit power which is identical in all sub-bands, $C_{m,s}$ represents its spreading sequence (chip by chip multiplication of the user assigned WH sequence and the cell specific scrambling code).

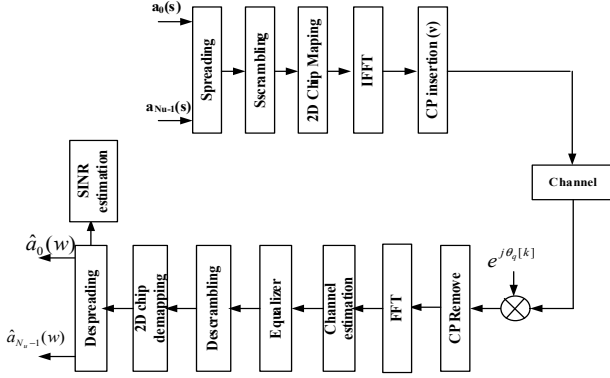


Figure 1: OFDM-CDMA Transceiver.

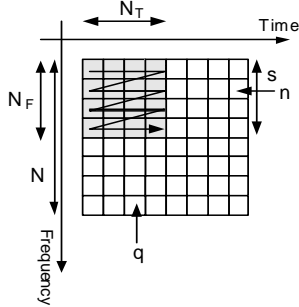


Figure 2: Time Frequency grid

At the output of the IFFT, a cyclic prefix CP of v samples is inserted to discard the inter symbol interference. The signal is then converted to Radio Frequency by the RF unit. It is then convoluted by the multipath propagation channel which is modelled by its base band function as a FIR filter $h_q(z) = \sum_{k=0}^{W-1} g_q[k]z^{-k}$ where $g_q[k]$ is the Channel Impulse Response (CIR) and W is the delay spread which is assumed less than the cyclic prefix ($W \leq v$). The signal at the channel output is then corrupted by an AWGN with variance σ^2 .

At the receiving side, the signal is down converted by the receiver RF unit. The phase noise is the phase error between the carrier used for up conversion at the transmitter and the carrier used for down conversion at the receiver. It can be represented by a simple random phase rotation of the samples at the input of the CP remove module. Therefore, the received samples can be written as

$$r_q[k] = \frac{e^{j\theta_q[k]}}{N} \sum_{s=0}^{S-1} \sum_{n=0}^{N_F-1} b_q[sN_F + n] h_q[sN_F + n] e^{-j2\pi \frac{sN_F + n}{N} k} + n_q[k] \quad (2)$$

with $k = 0, \dots, N - 1$

Where $h_q[sN_F + n] = h_q(z) \big|_{z=e^{j2\pi \frac{sN_F + n}{N}}}$ is the FFT of the CIR $g_q[k]$ on the sub carrier n of s^{th} sub band, $n_q[k]$ is the AWGN and $\theta_q[k]$ is the phase noise.

After the FFT operation, the value of $(wN_F + p)^{\text{th}}$ sub-carrier of the q^{th} OFDM symbol of an OFDM-CDMA block is:

$$R_q[wN_F + p] = \sum_{s=0}^{S-1} \sum_{n=0}^{N_F-1} b_q[sN_F + n] \phi(w, s, p, n, q) + N_q[wN_F + p] \quad (3)$$

w is the desired sub-band index ($w = 0, \dots, S-1$) and p is the index of a sub-carrier in the w^{th} sub-band ($p = 0, \dots, N_F-1$). $\phi(w, s, p, n, q)$ is the "frequency side-to-side" equivalent channel transfer function (IFFT-channel-FFT) including the effect of the phase noise. It is given by

$$\phi(w, s, p, n, q) = \frac{h_q[sN_F + n]}{N} \sum_{u=0}^{N-1} \exp\left(-j2\pi \frac{(w-s)N_F + (p-n)}{N} u\right) \exp(j\theta_q[u]) \quad (4)$$

Equation (4) shows that for $w=s$ and $p=n$ (interesting $(wN_F + p)^{\text{th}}$ sub-carrier), $\phi(w, w, p, p, q)$ characterizes the CPE altering equivalently all sub carriers.

Without loss of generality, we assume that one is interested by the symbols of user 0. In order to write the received signal with a matrix-vector notation, the following matrices are defined: $P = \text{diag}(P_0, \dots, P_{N_u-1})$ is the $N_u \times N_u$ diagonal matrix which entries are the powers allocated to each user, $Q = \text{diag}(P_1, \dots, P_{N_u-1})$ is the $(N_u-1) \times (N_u-1)$ diagonal matrix containing the powers of the interfering users, $C[s] = (C_0[s], C_1[s], \dots, C_{N_u-1}[s])$ is the $N_c \times N_u$ matrix containing all the spreading codes used in the s^{th} sub-band and $U[s] = (C_1[s], \dots, C_{N_u-1}[s])$ is the $N_c \times (N_u-1)$ matrix containing the codes of the interfering users in the s^{th} sub-band. Both matrices $C[s]$ and $U[s]$ depend on sub-band index s because of the long scrambling code. We also define the vectors $a[s] = (a_0[s], \dots, a_{N_u-1}[s])^T$ and $\tilde{a}[s] = (a_1[s], \dots, a_{N_u-1}[s])^T$ corresponding to the symbols of all users and interfering users respectively transmitted in the s^{th} sub-band. The estimated symbol of the reference user on the sub-band w after equalization and despreading is then:

$$\begin{aligned} \hat{a}_0[w] &= I_0 + I_1 + I_2 + I_3 + I_4 \\ I_0 &= \sqrt{P_0} C_0^H[w] Z[w] H[w, w] C_0[w] a_0[w] \\ I_1 &= C_0^H[w] Z[w] H[w, w] U[w] Q[s] \tilde{a}[w] \\ I_2 &= \sum_{\substack{s=0 \\ s \neq w}}^{S-1} C_0^H[w] Z[w] H[w, s] C[s] P[s] a[s] \\ I_3 &= C_0^H[w] Z[w] N[w] \end{aligned} \quad (5)$$

I_0 represents the useful signal, I_1 the Multiple Access Interference (MAI) of the same sub band w , I_2 the interference generated by all users from other sub bands (Inter Band Interference IBI) and I_3 the noise. $H[w, s]$ is a $N_c \times N_c$ matrix which components represent the frequency channel coefficients depending on the phase noise.

$$H_{p,n}[w, s] = \begin{pmatrix} A[0,0] & \dots & \dots & A[0, N_F-1] \\ A[1,0] & \ddots & & A[1, N_F-1] \\ \vdots & \vdots & A[p,n] & \vdots \\ A[N_F-1,0] & \dots & \dots & A[N_F-1, N_F-1] \end{pmatrix} \quad (6)$$

$$A[p,n] = \begin{pmatrix} \phi(w, s, p, n, 0) & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \phi(w, s, p, n, N_T-1) \end{pmatrix}$$

$Z[w]$ is a $N_c \times N_c$ diagonal matrix which equalizes the diagonal elements of $H[w, s]$.

III. PHASE NOISE MODEL

The phase noise $\theta(t)$ generated at the carrier oscillator can be described as a continuous Brownian motion process with a zero mean and variance $2Dt$. D is the phase noise line-width commonly known as the diffusion factor [12]. It characterizes the 3-dB bandwidth of its Lorentzian Power Spectral Density

function [3]. Such noise has independent Gaussian increments which can be represented as a finite-power Wiener phase whose spectrum decreases with the inverse of the square of the frequency. Its PSD is

$$S_\theta(f) = \frac{2D}{D^2 + (2\pi f)^2}$$

The phase noise $\theta_q[k]$ given in the previous equations can be obtained from $\theta(t)|_{t=[q(N+v)+k]T_s}$ (T_s is the sampling period) with the iterative function

$$\theta_q[k+1] = \theta_q[k] + \Gamma_q[k] \quad (7)$$

Where $\Gamma_q[k]$ is a zero mean i.i.d gaussian random variable with variance $2DT_s$. When the oscillator is integrated in a PLL, its Wiener phase noise is filtered by the PLL closed loop filter $F(z)$. Therefore, the filtered phase noise PSD is

$$S(f) = S_\theta(f) |F(e^{j2\pi f})|^2 \quad (8)$$

In this case (the free-running oscillator is suited by a Phase Locked Loop synthesizer), the phase noise is assumed as the so-called small-angle approximation. Moreover, for small-angle assumption, (4) yields:

$$\phi(w, s, p, n, q) = h_q[sN_F + n] \left(\delta(w, s)(p, n) + \frac{j}{N} \sum_{u=0}^{N-1} \theta_q[u] \exp\left(-j2\pi \frac{(w-s)N_F + (p-n)u}{N}\right) \right) \quad (9)$$

Where $\delta(w, s)(p, n) = 1$ if $w=s$ and $p=n$ and zero elsewhere. The assumption of small angle is due to the reduction of the variance of the phase noise by introducing it in a PLL filter.

In this paper, the PLL synthesizer is modelled by a second order high pass filter. Its coefficients are synthesized according to the cut-off frequency and the resonance parameter of the filter. It is given

$$F(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}} \quad (10)$$

IV. SINR EVALUATION

The data symbols are assumed i.i.d. having zero mean and unit variance. The SINR for every sub band is deduced from (5) by calculating the expectation values over the random data symbols of I_0 , I_1 , I_2 and I_3 . It is given by

$$\text{SINR} = \frac{E[I_0]^2}{E[I_1]^2 + E[I_2]^2 + E[I_3]^2} \quad (11)$$

The expectations values in (11) are given by

$$\begin{aligned} E[I_0]^2 &= P_0^2 C_0^H[w] Z[w] H[w, w] C_0[w] \\ E[I_1]^2 &= C_0^H[w] Z[w] H[w, w] U[w] Q U^H[w] H[w, w] Z[w]^H C_0^H[w] \\ E[I_2]^2 &= \sum_{s=0}^{S-1} C_0^H[w] Z[w] H[w, s] C[s] P C^H[s] H[w, s]^H Z[w]^H C_0^H[w] \\ E[I_3]^2 &= \frac{\sigma^2}{N_c} \text{tr}(Z[w] Z[w]^H) \end{aligned} \quad (12)$$

These expressions show that the SINR depends on a complex way of the actual spreading codes. (12) can not be used practically due to its complexity. Thus, we define some properties of random matrix and free probability theories as in our previous works [8][9][10] to show that the dependence of the SINR on the spreading codes was vanishing in the asymptotic regime (N_c and $N_u \rightarrow \infty$ while the ratio $\alpha = N_u/N_c$ is kept constant).

Property 1: if A is a $N_c \times N_c$ uniformly bounded deterministic matrix and

$C_m = \frac{1}{\sqrt{N_c}} (c_m(0), \dots, c_m(N_c - 1))$ where $c_m(l)$ are iid complex

random variables with zero mean, unit variance and finite eighth order moment, then [18]:

$$C_m^H A C_m \xrightarrow[N_c \rightarrow \infty]{} \frac{1}{N_c} \text{tr}(A) \quad (13)$$

This property is used to evaluate $E[I_0]^2$, $E[I_1]^2$, and $E[I_2]^2$.

Property 2: Let C a Haar distributed unitary matrix of size $N_c \times N_u$ [17]. C can be decomposed into a vector C_0 of size N_c and a matrix U of size $N_c \times N_{u-1}$. So, C can be written as $C = (C_0, U)$. Given these hypotheses, it is proven in [16] that:

$$U Q U^H \xrightarrow[N_c \rightarrow \infty]{} \alpha \bar{P} (I - C_0 C_0^H) \quad (14)$$

$\alpha = N_u/N_c$ is the system load and $\bar{P} = \frac{1}{N_u - 1} \sum_{m=1}^{N_u-1} P_m$ is the

average power of the interfering users. The Haar distributed assumption is only technical and will not change in the total result. The simulation results obtained in [16] with a Walsh-Hadamard spreading matrix match well with the theoretical model. This lemma is used to evaluate $E[I_1]^2$ and $E[I_2]^2$.

Property 3: If C is generated from a $N_c \times N_c$ Haar unitary random matrix then matrices $C[s] Q C^H[s]$ and $Z[w]^H H[w, s] H[w, s]^H Z[w]$ are asymptotically free almost everywhere [19]. Then, one can conclude:

$$\begin{aligned} \frac{1}{N_c} \text{tr}(Z[w] H[w, s] C[s] P C^H[s] H[w, s]^H Z[w]^H) &\xrightarrow[N_c \rightarrow \infty]{} \\ \frac{1}{N_c} \text{tr}(Z[w] H[w, s] H[w, s]^H Z[w]^H) \times \frac{1}{N_c} \text{tr}(C[s] P C^H[s]) &\quad (15) \end{aligned}$$

This lemma is used to evaluate $E[I_2]^2$. With these assumptions, (12) yields for a given phase noise value [9][10]:

$$\begin{aligned} E[I_0]^2 &= P_0^2 \left| \frac{1}{N_c} \text{tr}(Z[w] H[w, w]) \right|^2 \\ E[I_1]^2 &= \alpha \bar{P} \left(\frac{1}{N_c} \text{tr}(Z[w] H[w, w] H[w, w]^H Z[w]^H) - \left| \frac{1}{N_c} \text{tr}(Z[w] H[w, w]) \right|^2 \right) \\ E[I_2]^2 &= \frac{\alpha \bar{P}}{N_c} \sum_{s=0}^{S-1} \text{tr}(Z[w] H[w, s] H[w, s]^H Z[w]^H) \\ E[I_3]^2 &= \frac{\sigma^2}{N_c} \text{tr}(Z[w] Z[w]^H) \end{aligned} \quad (16)$$

We will now exploit (16) to expand an analytical expression of the asymptotical SINR. First, we assume that the receiver uses a MMSE equalizer which coefficients are given by:

$$z_q[w N_F + p] = \frac{\phi^*(w, w, p, q)}{|\phi(w, w, p, q)|^2 + \gamma}$$

γ is the inverse of the SNR per sub-band : $\gamma = \frac{\alpha \bar{P}[w]}{\sigma^2}$.

Notice that, according to equation (4), the coefficient $\phi(w, w, p, q)$ resumes to the time average during one OFDM symbol of the equivalent channel. Expanding the expressions of (16) and computing the expectations over the phase noise realizations, we use Parseval's identity to verify that (16) yields

$$\begin{aligned}
E[I_0]^2 &= E_0 \left[\frac{1}{N_c} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_F-1} \frac{|h_q[wN_F+p]|^2}{|h_q[wN_F+p]|^2 + \gamma} \right]^2 \\
I[w] &= \frac{\alpha \bar{P}}{N_c} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_F-1} \frac{|h_q[wN_F+p]|^4}{(|h_q[wN_F+p]|^2 + \gamma)^2} - \alpha \bar{P} \left[\frac{1}{N_c} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_F-1} \frac{|h_q[wN_F+p]|^2}{|h_q[wN_F+p]|^2 + \gamma} \right]^2 \\
&+ \frac{\sigma^2}{N_c} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_F-1} \frac{|h_q[wN_F+p]|^2}{(|h_q[wN_F+p]|^2 + \gamma)^2} \\
&+ \frac{\alpha \bar{P}}{N_c} \sum_{p=0}^{N_F-1} \sum_{n=0}^{N_F-1} \sum_{q=0}^{N_F-1} \frac{|h_q[wN_F+p]|^2 |h_q[wN_F+n]|^2}{(|h_q[wN_F+p]|^2 + \gamma)^2} \int_{-1/2}^{1/2} S(f) \left| \psi_N \left(\frac{p-n}{N} - f \right) \right|^2 df \\
&+ \frac{\alpha \bar{P}}{N_c} \sum_{s=0}^{S-1} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_F-1} \sum_{n=0}^{N_F-1} \frac{|h_q[wN_F+p]|^2 |h_q[sN_F+n]|^2}{(|h_q[wN_F+p]|^2 + \gamma)^2} \int_{-1/2}^{1/2} S(f) \left| \psi_N \left(\frac{(w-s)N_F + p-n}{N} - f \right) \right|^2 df
\end{aligned} \tag{17}$$

Where $I[w] = E[I_1]^2 + E[I_2]^2 + E[I_3]^2$ is the total interference power and Ψ_N is the function defined by $\psi_N(x) = \frac{1}{N} \frac{\sin(\pi Nx)}{\sin(\pi x)}$.

V. WEIGHTING FUNCTIONS

Equation (17) shows that the interference power is composed of three expressions which are independent on the phase noise and two others which are dependent on the phase noise. A better understanding of the phase noise is clearly needed. Therefore, we will define the weighting functions deduced from (17) (as given by Stott for OFDM [4] and [7] for MC-CDMA) for a Gaussian channel by

$$\begin{aligned}
W_1(f) &= \frac{1}{N_F} \sum_{p=0}^{N_F-1} \sum_{n=0}^{N_F-1} \left| \psi_N \left(\frac{p-n}{N} - f \right) \right|^2 \\
W_2(f) &= \frac{1}{N_F} \sum_{s=0}^{S-1} \sum_{p=0}^{N_F-1} \sum_{n=0}^{N_F-1} \left| \psi_N \left(\frac{(w-s)N_F + p-n}{N} - f \right) \right|^2
\end{aligned} \tag{18}$$

Where $W_1(f)$ represents the effect of ICI within the w^{th} sub band and $W_2(f)$ represents the effect of IBI on the w^{th} sub band. We can verify that (18) can be written as

$$W_1(f) = \begin{cases} \frac{N_F}{N_F} f + \frac{N_F+1}{N_F} & \text{if } \frac{-N_F-1/2}{N} \leq f \leq \frac{-1}{2N} \\ 0 & \text{if } \frac{-1}{2N} \leq f \leq \frac{1}{2N} \\ \frac{2N_F+1}{1-2N_F} \left(\frac{N_F}{N_F} f - 1 \right) & \text{if } \frac{1}{2N} \leq f \leq \frac{N_F+1/2}{N} \end{cases}$$

$$W_2(f) = \begin{cases} \frac{2N_F+1}{2N_F-1} \left(\frac{N_F}{N_F} f + 1 \right) & \text{if } \frac{-SN_F}{N} \leq f \leq \frac{-(S-1)N_F-1/2}{N} \\ 1 & \text{if } \frac{-(S-1)N_F-1/2}{N} \leq f \leq \frac{-2N_F+1}{2N} \\ \frac{2N_F+1}{1-2N_F} \frac{N_F}{N_F} f & \text{if } \frac{-2N_F+1}{2N} \leq f \leq \frac{-1}{2N} \\ 0 & \text{if } \frac{-1}{2N} \leq f \leq \frac{1}{2N} \\ \frac{2N_F+1}{2N_F-1} \frac{N_F}{N_F} f & \text{if } \frac{1}{2N} \leq f \leq \frac{N_F-1/2}{N} \\ 1 & \text{if } \frac{N_F-1/2}{N} \leq f \leq \frac{(S-1)N_F+1/2}{N} \\ \frac{2N_F+1}{1-2N_F} \left(\frac{N_F}{N_F} f + S \right) & \text{if } \frac{(S-1)N_F+1/2}{N} \leq f \leq \frac{SN_F+1/2}{N} \end{cases} \tag{19}$$

With these assumptions, the interference power in (17) can be written for a Gaussian channel as:

$$I[w] = \frac{\alpha \bar{P}}{(1+\gamma)^2} \int_{-1/2}^{1/2} S(f) W_1(f) df + \frac{\alpha \bar{P}}{(1+\gamma)^2} \int_{-1/2}^{1/2} S(f) W_2(f) df + \frac{\sigma^2}{(1+\gamma)^2} \tag{20}$$

Where the first term indicates the ICI interference within the band w , the second term indicates the IBI and the third term is the AWGN. The weighting functions given in Figure 3 and Figure 4 versus the normalized frequency f (to sub-carrier spacing) combined with equation (20) are of great help in visualizing the impact of the filtered phase noise on the studied sub band. Figure 3 shows that the low frequency components of the phase noise PSD have more impact on the

ICI however, Figure 4 shows that all frequency components of the phase noise PSD have an equivalent impact on the IBI. Thus, to discard the phase noise effect, the engineer designers have to chose PLL filters with **higher cut-off frequency**. Moreover, a PLL cut-off frequency **greater** than the sub-carrier spacing will eliminate definitively the ICI while the IBI obstinately persists.

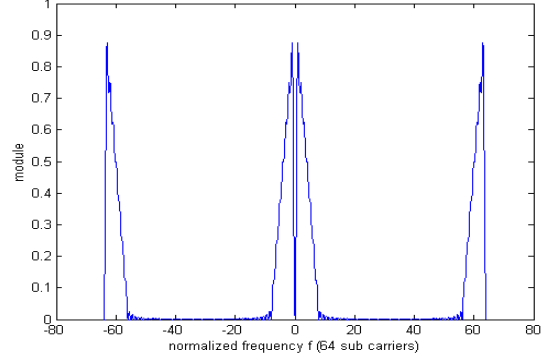


Figure 3: weighting function $W_1(f)$

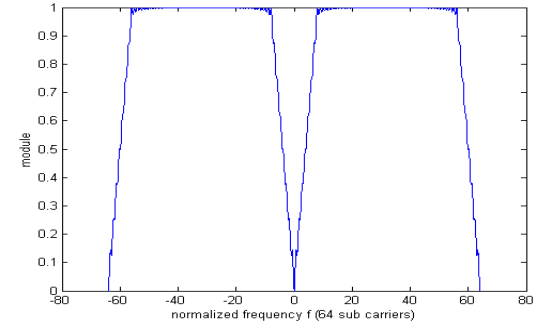


Figure 4: weighting function $W_2(f)$

VI. SYSTEM PERFORMANCE

In this section, we will validate our theoretical model given previously by means of simulations. We first compare the SINR computed with (11) and (17) with the instantaneous SINR measured via Monte Carlo simulations. However, for a frequency selective slow fading physical channel and due to the variations of the channel caused by the mobile speed, we give the comparison between the average SINRs. The simulations assumptions in the sequel are the followings:

- FFT size: $N=64$, Spreading factor: $N_c = 32$ chips, QPSK modulation, sub-carrier spacing=312.5KHz.
- Scrambling code: concatenation of 19 Gold codes of 128 chips each.
- Spreading schemes: MC-CDMA: ($N_F=32$, $N_T=1$), OFDM-CDMA: ($N_F=8$, $N_T=4$), MC-DS-CDMA: ($N_F=1$, $N_T=32$).
- PLL filter (CutOff=100KHz): $b=[0.9770 \ -1.9541 \ 0.9770]$ and $a=[1 \ -1.9881 \ 0.9882]$.
- PLL filter (CutOff=1MHz): $b=[0.9256 \ -1.8509 \ 0.9256]$ and $a=[1 \ -1.8790 \ 0.8876]$.

Figure 5 illustrates the comparison between theoretical and simulated average SINRs for BRAN A channel [20], a mean ratio $E_b/N_0=20\text{dB}$ and a PLL cutoff frequency of 1MHz. The SINRs have been measured in the first sub-band ($w=0$). Figure 5 shows that our theoretical model matches

perfectly with simulations, even for a relatively small spreading factor ($N_c=32$). Moreover, it shows that the SINR sensitivity to phase noise becomes noticeable for a diffusion factor $D>1\text{KHz}$.

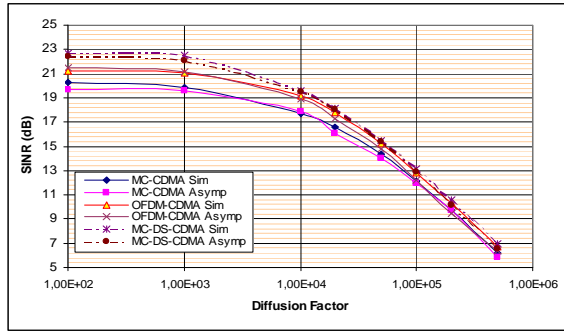


Figure 5- validation of theoretical model (BRAN A).

Since the theoretical model has been validated, we will exploit equation (17) to give more insights of the phase noise. Thus, we define the instantaneous degradation by $De_{g(dB)} = 10 \times \log_{10} \left(\frac{SINR_{max}}{SINR} \right)$ where $SINR_{max}$ is the SINR obtained for a perfectly synchronized system.

Figure 6 and Figure 7 give the comparison between degradations of different spreading schemes to carrier phase noise for a BranA channel model, a mean ratio $E_b/N_0=20\text{dB}$, a full load and a quarter load respectively. It is easy to conclude that the sensitivity of the 3 spreading schemes to phase noise is similar. Moreover, the degradation increases with system load and decreases with the PLL cut off frequency.

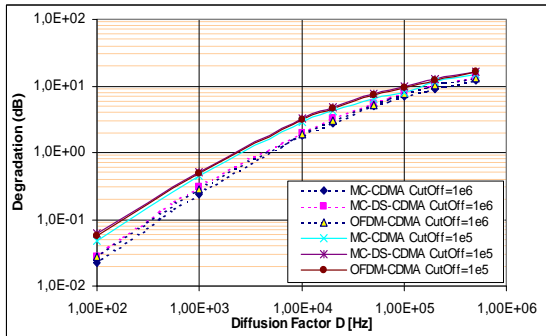


Figure 6: comparison between degradation of different spreading schemes

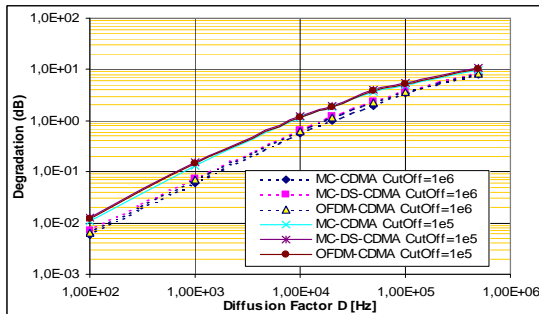


Figure 7: comparison between degradation of different spreading schemes

VII. CONCLUSION

In this article, we have investigated the effect of the phase noise on the performance of 2D OFDM-CDMA spreading schemes. We have determined an analytical expression of the SINR which is independent of the spreading codes and

showed that its results are conformed to Monte Carlo simulations. Also, we presented helpfully weighting functions used by the RF engineer to design efficient frequency synthesizers (a PLL with higher cutoff frequency is more efficient) and showed that MC-CDMA, OFDM-CDMA and MC-DS-CDMA spreading schemes are identically sensitive to the phase noise.

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